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FORCED OSCILLATIONS AND THE RADIATION OF SOUND BY A CIRCULAR PLATE INTERACTING WITH A FLUID*

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A method is proposed for calculating forced oscillations and the acoustic radiation of a circular disc during its axially symmetric oscillations in an infinitely rigid baffle on the boundary of separation between fluid media. The dependence of the components of the deflection and the acoustic pressure on the excitation frequency as well as their distribution over the surface of the plate are investigated.

The proposed method is simpler than the use of expansions over orthogonal systems of functions /1, 2/. It leads to a finite resolvent system which contains the values of the acoustic pressure at a series of fixed points on the surface of the disc as unknowns. Compared with the finite-difference method** (Golovanov V.A., Muzychenko V.V., Peker F.N. and Popov A.L., Scattering and sonic emission by elastic shells in a fluid, Preprint No.261, Inst. Problem Mekhaniki Akad. Nauk SSSR, Moscow, 70 pp., 1985.) the proposed method enables one to attain the required accuracy using a smaller number of mesh points and leads to resolvent systems with better computational properties (according to the conditionality index). We also remark upon a method for determining the displacement potential of the fluid using a function of the deflection of the non-axially symmetrically oscillating disc /3/ and the results of experimental investigations of the hydroelastic oscillations of a disc /4, 5/.

Consider the forced oscillations of a circular disc which is clamped in an infinitely rigid baffle on the boundary of separation between fluid half spaces. Omitting the time factor $\exp(-i\omega t)$, we shall write the equation for the flexure of the disc taking account of the reaction of the acoustic media in the form

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$$D\nabla^2\nabla^2 w - m\omega^2 w = q(r) + p(r), \quad \nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} \tag{1}$$

where w is the deflection of the disc, D and m are its cylindrical rigidity and linear mass, $q(r)$ is the excitation load and $p(r)$ is the sonic pressure on the disc (the difference between the pressures on its two sides).

We shall seek a solution of Eq.(1), which satisfies the condition that the deflection on the contour of the disc must be equal to zero, in the form of a series in Bessel functions

$$w = \Sigma_k w_k J_0(x_k r/a) \tag{2}$$

where x_k are the roots of the equation $J_0(x) = 0$, a is the radius of the disc and Σ_k denotes summation over k from 1 to ∞.

In order to satisfy the remaining boundary condition, we apply a bending moment M to the contour of the disc. The total load on the disc will therefore consist of the excitation load $q(r)$, the sonic pressure $p(r)$ and also a load with an intensity q' which acts over the area of an annulus of thickness ε_1 located at a distance ε from the edge of the disc. In order to pass to the bending moment it is necessary to assume /6/

$$q'\varepsilon_1 \underset{\varepsilon_1\to 0}{\longrightarrow} p', \quad p'\varepsilon \underset{\varepsilon\to 0}{\longrightarrow} M \tag{3}$$

Let us now expand the above-mentioned components of the transverse load in a series of Bessel functions. In doing this, we approximate the unknown function $p(r)$ in the interval $[0, a]$ by a piecewise linear function. By making use of the condition of orthogonality of Bessel functions, taking account of formula (3) and the approximation adopted for $p(r)$, we find the coefficients of the expansions for the above-mentioned loads

$$q_k = \frac{2}{a^2 J_1^2(x_k)} \int_0^a q(r) J_0\left(x_k \frac{r}{a}\right) r\,dr \tag{4}$$

$$q_k' = -\frac{2M}{a^2}\frac{x_k}{J_1(x_k)}, \quad p_k = \Sigma_j p_j z_{jk}$$

$$z_{jk} = \frac{2}{a^2 J_1^2(x_k)} \left[\int_{(j-1)\Delta r}^{j\Delta r} \frac{j\Delta r - r}{\Delta r} J_0\left(x_k \frac{r}{a}\right) r\,dr - \int_{(j-2)\Delta r}^{(j-1)\Delta r} \frac{(j-2)\Delta r - r}{\Delta r} J_0\left(x_k \frac{r}{a}\right) r\,dr \right]$$

$$j = 2, 3, \ldots, N; \quad k = 1, 2, \ldots$$

Here N is the number of segments into which the disc is subdivided, p_j is the sonic pressure at the mesh points and Σ_j denotes summation over j from 1 to $N+1$. The expression for z_{jk} when $j = 1$, $j = N+1$ are obtained from the general formula after discarding the second and first integrals, respectively.

Substitution of the expansions (2) and (4) into Eq.(1) enables one to find the coefficients

$$w_k = \frac{1}{m(\omega_k^2 - \omega^2)} \left[q_k - \frac{2M}{a^2}\frac{x_k}{J_1(x_k)} + \Sigma_j p_j z_{jk} \right], \quad \omega_k^2 = \frac{x_k^4}{a^4}\frac{D}{m}. \tag{5}$$

The bending moment M is found from the second condition on the contour. By considering the case of restraint $dw/dr|_{r=a} = 0$, we obtain

$$M = \frac{a^2}{2}\frac{Q_1 + \Sigma p_j s_j}{Q_2}, \quad Q_1 = \Sigma_k \frac{q_k x_k J_1(x_k)}{\omega_k^2 - \omega^2}$$

$$Q_2 = \Sigma_k \frac{x_k^2}{\omega_k^2 - \omega^2}, \quad s_j = \Sigma_k \frac{z_{jk} x_k J_1(x_k)}{\omega_k^2 - \omega^2} \tag{6}$$

By substituting (6) into (5) and then into (2), we arrive at the following expression for the deflection of the disc:

$$w = \Sigma_k \frac{J_0(x_k r/a)}{m(\omega_k^2 - \omega^2)} \left[q_k - \frac{Q_1}{Q_2}\frac{x_k}{J_1(x_k)} + \Sigma_j p_j \left(z_{jk} - \frac{s_j x_k}{Q_2 J_1(x_k)} \right) \right] \tag{7}$$

The sonic pressure p_2 in the half space $z > 0$, which is the result of the emission of sound by the disc, is determined by the Huygens' integral /7/

$$p_2(x, y, z) = -\frac{\omega^2\rho_0}{2\pi}\iint\limits_{(S)} w(x', y')\frac{\exp(ik_0R)}{R}\,ds \qquad (8)$$
$$R = [(x'-x)^2 + (y'-y)^2 + (z'-z)^2]^{1/2}$$

where R is the distance from the element dS to the point of observation, S is the area of the disc, ρ_0 is the density of the medium, and k_0 is the wave number. The pressure p_1 in the half space $z<0$ is written out in a similar manner. By transforming (8) to cylindrical coordinates r, θ and z, we get the expression

$$p_l = \frac{\omega^2\rho_0 n}{2\pi}\int_0^a\int_0^{2\pi} w(\rho)\frac{\exp(ik_0R_l)}{R_l}\rho\,d\theta\,d\rho \qquad (9)$$
$$R_l^2 = (l-1)^2\Delta r^2 + \rho^2 - 2(l-1)\Delta r\rho\cos\theta$$

for the difference in the pressures at the point $(r = (l-1)\Delta r,\ z = 0)$.

Here n is a parameter which is equal to 1 or 2 when the disc makes contact with the medium on one or two sides and the function $w(\rho)$ is defined by formula (7).

By putting $l = 1, 2, \ldots, N+1$ in (9), we arrive at the system of algebraic equations for finding the values of the sonic pressure at the mesh points on the surfaces of the disc

$$\Sigma_j a_{lj}p_j = b_l, \quad l = 1, 2, \ldots, N+1 \qquad (10)$$

$$a_{lj} = \Sigma_k \frac{z_{jk}I_{lk}}{\omega_k^2-\omega^2} - \frac{s_j}{Q_2}\Sigma_k\frac{x_kI_{lk}}{(\omega_k^2-\omega^2)J_1(x_k)} - \frac{2\pi m}{\omega^2\rho_0 n}\delta_{lj} \qquad (11)$$
$$b_l = -\Sigma_k\frac{q_kI_{lk}}{\omega_k^2-\omega^2} + \frac{Q_1}{Q_2}\Sigma_k\frac{I_{lk}x_k}{J_1(x_k)(\omega_k^2-\omega^2)}$$
$$\delta_{lj} = \begin{cases}1, & l = j\\ 0, & l \neq j\end{cases}$$
$$I_{lk} = \int_0^a\int_0^{2\pi} J_0\left(x_k\frac{\rho}{a}\right)\frac{\exp(ik_0R_l)}{R_l}\rho\,d\theta\,d\rho$$

By separating out the real and imaginary parts in (10) and solving the system of $2(N+1)$ equations resulting from this, we find the real and imaginary components of the sonic pressure at the nodes. Next, using formula (7), we calculate the deflection of the disc at the necessary points and the acoustic power of the radiation using the formula

$$N = \frac{\pi}{n}\operatorname{Re}\int_0^a pv^*r\,dr \qquad (12)$$

(v^* is the complex conjugate of the vibrational velocity).

We note that, when $\omega \to \omega_k$, the solution of system (10) remains finite since terms which contain the difference $\omega_k^2 - \omega^2$ in the denominator occur both in the coefficients of the system as well as on its right-hand sides. The deflection of the disc, which is determined by formula (7), also remains finite since, in the corresponding term, the numerator and the denominator simultaneously tend to zero.

In the calculation it is necessary to omit the terms which go to infinity at the frequencies $\omega = \omega_k$ $(k = 1, 2, \ldots)$ in the sums (11) and to introduce the additional unknown quantity w_k into system (10), the coefficients accompanying which will be the integrals I_{lk}. The corresponding additional equation is obtained by equating the expression in the square brackets of formula (5) to zero. However, it is considerably simpler to carry out (starting from the continuity and boundedness of the functions $p(r)$ and $w(r)$) an interpolation of the results obtained close to the frequency ω_k. We now present the results of a calculation of the forced oscillations and sonic emission of a circular steel disc of radius $a = 0.6$ m and with a thickness $h = 10^{-2}$ m which is excited at the centre by a concentrated force $F = 10$ H The integrals I_{lk} in (11) were calculated using Gaussian quadrature formulae, the number of segments into which the disc was subdivided was varied from 15 to 20 and the number of terms in the series was varied from 10 to 15. A further increase in these parameters had hardly any effect on the results over the range of variation of the excitation frequency which was considered.

The dependences of the real and imaginary components of the deflection at the centre of the disc on the excitation frequency are shown by curves *1* and *2* in Fig.1 for the case of one-sided contact of the disc with water. The analogous dependences for the components of the sonic pressure at the centre are shown in Fig.2. It can be seen that, at certain excitation frequencies, the bending modulus and the sonic pressure modulus and their imaginary parts reach a maximum value while the real parts change sign. This corresponds to the first two resonances of the disc which, in the case of oscillations in vacuo are observed at the frequencies $\omega_1 = 434\ s^{-1}$ and $\omega_2 = 1700\ s^{-1}$.

As might have been expected, the presence of a fluid leads to a displacement of the resonances of the disc towards lower frequencies. The distribution of the deflections over the radius of the disc at the first and second resonances is barely distinguishable from the corresponding forms of the oscillations in vacuo. At the higher resonance forms of the oscillations this difference becomes more noticeable and, in the case of the third resonance, it is illustrated in Table 1 which shows the ratios $(w(r)/w(0))\cdot 10^3$ for the following versions: a) for oscillations in vacuo, b) when there is one-sided contact with water and $h/a = 1/30$, c) as in b) but with $h/a = 1/120$.

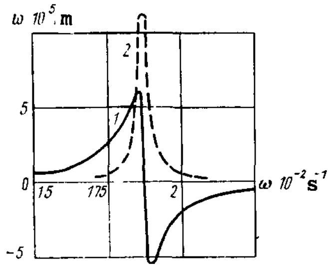

Fig.1

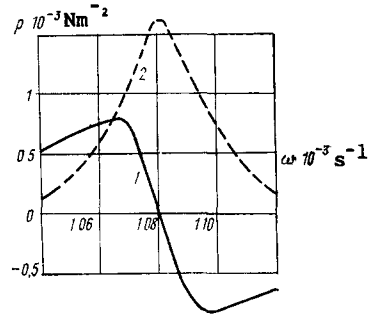

Fig.2

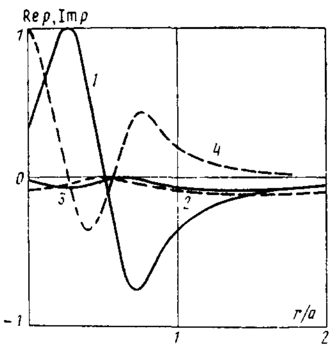

Fig.3

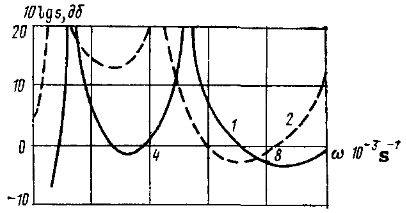

Fig.4

Table 1

Variant	r/a=0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
a	813	280	—198	—401	—262	52	270	286	110
b	780	282	—205	—396	—254	60	267	252	67
c	780	284	—216	—435	—322	—12	220	238	67

It can be seen from the results which have been presented that the lack of agreement between the forms of the resonance oscillations and the natural vibrational modes in vacuo is more pronounced at small relative thicknesses of the disc. Meanwhile, it is not so considerable as that which was noted in /8/ in the case of a strip plate which is explained by the localization of the higher resonant oscillations close to the centre of the disc. For comparison, we point out that, at the third resonance of a freely supported strip with a ratio $h/l = 1/50$, the amplitude of the halfwave at $x = l/6$ is approximately 0.8 of the amplitude when $x = l/2$.

The sonic pressure distribution on the surface of the plate repeats, as a whole, the deflection distribution, but its value is non-zero on the contour and then further decays upon becoming more remote from the edge of the plate. What has been said above is illustrated in Fig.3 where the dependences of $\mathrm{Re}\, p$ and $\mathrm{Im}\, p$ on the ratio r/a at the non-resonant frequency $\omega = 2000\ s^{-1}$ (curves *1* and *2*) and the third resonant frequency $\omega = 2846\ s^{-1}$(curves *3* and *4*) are shown. The values of the pressure components are reduced with respect to the maximum value of one of them (the real part at the non-resonant frequency and the imaginary part at the resonant frequency).

The frequency dependence (curve *1*) of the coefficient

$$s = N/N_0,\ N_0 = F^2\rho_0/(4\pi m^2 c_0)$$

of a circular steel disc of thickness $h = 0.03$ m and radius $a = 0.6$ m during its oscillations in air under the action of a concentrated force at the centre, is shown in Fig.4. The quantity N_0 is the acoustic power of the radiation due to an infinite disc which is excited by a low-frequency concentrated force ($\omega \ll \omega_*$, where ω_* is the limit frequency of the plate /7/). An analogous curve *2* was obtained in the case of single-sided contact with water ($N_0 = F^2\omega^2/(12\pi\rho_0 c_0^3)$).

The results of the calculations show that the finite nature of the dimensions of the disc and the conditions under which it is clamped have an effect on the appearance and the position of the emission resonance maxima. As the frequency increases outside of the resonance zones, the acoustic emission powers of bounded and infinite discs approach one another and this takes place more rapidly in the case of a disc which radiates in air. We also note that, with the exception of the resonance frequencies, the effect of the aerial acoustic field on the oscillations of a disc is negligibly small.

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